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Part 1: Introduction

The game “ANGRY BIRDS”

The game Angry Birds created by Rovio entertainment and it was released in December, 2009. It was a really popular game for everybody back then. The point of the game is to score a very high score to unlock new level, to do that we have to destroy all “green pig(s)” that hides in a bunker made out of glasses or woods or stones or a combination of all. And to destroy the bunker and the pigs, we have to find a weak spot of the bunker and launch some bird(s) that are provided for us from a sling shot to strike it. Each fewer birds we use, 10000 points we earn. That means we have to use as few birds as possible to kill all the pigs.

There are many levels that are very hard to beat due to the distance from the slingshot and the target and the obstacles. There are so many angry bird math projects out there on the internet, but most of them are based on the red bird which travel in parabolic path to strikes targets like a classic projectile cannon.

Later on in the game, more birds were introduced to the player. There is a yellow bird called “Chuck” and a boomerang bird called “Hal,” they are very hard to be used because we have activate their special powers at certain point which is very hard to time it right; without using the special power and strike with a usual parabolic path, the damaged caused is slightly less than the red bird. If we use them right it will be very effective. I haven't seen the Yellow bird and Boomerang bird math project yet.
The Reason for This Topic

Back then when I used to play Angry Birds, I found the game very addictive, sometimes I got stuck on one level for too long because sometimes the flight path of the bird was way too short or the layout of the base was too strong. I got very frustrated especially when I was using the bird with special powers. I always triggered the super power at the wrong moment and could not get enough damage on the base and I had to start over.

From the love of the game combined with the math I have learned from IB math HL made me choose topic for this IA project. In our IB math HL we did calculus so, I would like to know how we use anti-derivative from the slope to find the real equation and I like how we use the derivative to find the slope of a new line on graph and connect it to the first line. For this project I will mainly use derivative to form a new function to solve the best point to activate the special powers.

Part 2: The Flight Path and Equation for Chuck

Chuck normally travels in a parabolic paths but when we click the screen, it will activate the Chuck’s boost mode. His flight path will change, he will be travel in the linear path to strikes the target with the slope of the derivative of the point on the parabola where the boost mode is activated or we could call it a tangent of the parabola. There is also the Angry Bird game available for a laptop, the great thing about it is that we are able to download the program that displays a graph grid on the screen. To find the flight path, I ran the program with the game and I set the location of slingshot with a bird at the point (0,0) that way, it is easier to do the calculation. So when I played the game, I could see where the bird land and where the x-intercepts were and also the location of the vertex. So I would be able to figure out the flight path of the birds.

The furthest distance traveled in the parabolic path without activating a boost in the horizontal direction is 30. I recorded the coordinated at 10 random points of the flight path of the yellow bird for each trial in the table below.

<table>
<thead>
<tr>
<th>Trial #1</th>
<th>x</th>
<th>0</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>22</th>
<th>25</th>
<th>26</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>1.16</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>8</td>
<td>7.04</td>
<td>5</td>
<td>4.16</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
The highest point of the graph or the vertex is point (15,9) and the two x-intercepts are point (0,0) and point (30,0). Now, we are one step closer to finding the first part of the flight path.

**Making an equation:**
The quadratic equation is the best way to represent the model.

\[ y = a( x - p )( x - q ) \]

Values of x when y = 0 are 0 and 30
Substitute both values of x for p and q; \[ y = a( x - 0 )( x - 30 ) \]
Solve for a
Substitute values of x and y at the vertex; \[ 9 = a( 15 - 0 )( 15 - 30 ) \]
\[ 9 = a(15)(-15) \]
\[ 9 = -225a \]
\[ a = -0.04 \]

equation : \[ y = -0.04x( x - 30 ) \]

Are the two equation the same? No, the equation that fits the model the most is equation #2. It is open down and has the same x-intercepts as the data recorded from the game.
Now we have the flight path before the boost mode. And now, we are looking for the path for the boost mode. First, we have find the slope of the tangent on the parabola. To find slope of tangent we have to find the derivatives of the parabola by using the first principle.

**method 1 :**

**equation :**

\[ y = -0.04x(x - 30) \]

**re-arranging:**

\[ y = -0.04x^2 + 1.2x \]

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{\{-0.04(x + h)^2 + 1.2(x + h)\} - \{-0.04x^2 + 1.2x\}}{h}
\]

\[
= \lim_{h \to 0} \{ -0.04x^2 - 0.08xh - 0.04h^2 + 1.2x + 1.2h + 0.04x^2 - 1.2x \}/h
\]

\[
= \lim_{h \to 0} \{ -0.08xh - 0.04h^2 + 1.2h \}/h
\]

\[
= \lim_{h \to 0} -0.08x - 0.04h + 1.2
\]

The value of \( h \) is very very close to zero

Substitute 0 in for \( h \) ; \n
\[ f'(x) = -0.08x + 1.2 \]

Equation of the slope of the tangent; \( y' = -0.08x + 1.2 \)

The point of boost activation can be any where in the region.

The dark blue lines are the examples of the tangent that could be formed in the red region.

The black points where the lines of tangent start were the activation point. The coordinate of it is \((x_0, y_0)\)
The Tangent Line Function state that: \( y - y_0 = f'(x_0) \cdot (x - x_0) \)

\((x_0, y_0)\) are the coordinates of the line where the boost mode will be activated so

\[ y_0 = -0.04x^2 + 1.2x \]

Substitute in the equation; \( y - (-0.04x_0^2 + 1.2x_0) = f'(x_0) \cdot (x - x_0) \)

\(f'(x_0)\) is the slope of \( f(x_0) = -0.04x_0^2 + 1.2x_0 \) which is \(-0.08x_0 + 1.2\)

Substitute in the slope; \( y - (-0.04x_0^2 + 1.2x_0) = (-0.08x_0 + 1.2) \cdot (x - x_0) \)

\(x\) and \(y\) are the value make up the coordinate of the point that we want to strike.

If we want to boost through the wood to strike the point \((27, 6)\) just to destroy the top of the pyramid without killing the pig. At what coordinate should we activate the boost?

\[ y - (-0.04x_0^2 + 1.2x_0) = (-0.08x_0 + 1.2) \cdot (x - x_0) \]

\[ 6 + 0.04x_0^2 - 1.2x_0 = (-0.08x_0 + 1.2) \cdot (27 - x_0) \]

\[ 6 + 0.04x_0^2 - 1.2x_0 = -2.16x_0 + 0.08x_0^2 + 32.4 - 1.2x_0 \]

\[ 6 + 0.04x_0^2 - 1.2x_0 = -3.36x_0 + 0.08x_0^2 + 32.4 \]

\[ 0 = -2.16x_0 + 0.04x_0^2 + 26.4 \]

Graph the equation \( y = -2.16x_0 + 0.04x_0^2 + 26.4 \) on the graph and find the x-intercepts

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
By using the quadratic formula:
a = 0.04, b = -2.16 and c = 26.4

\[ x_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x_0 = 27 \pm 8.307 \]
\[ x_0 = 18.693 \text{ and } 35.307 \]

Substitute \( x_0 \) in the \( f(x_0) = y_0 = -0.04x_0^2 + 1.2x_0 \)

\[ y_0 = -0.04(18.693)^2 + 1.2(18.693) \]
\[ y_0 = 8.454 \]

so the activation point is (18.693, 8.454)

Substitute \( x_0 \) in the \( f(x_0) = y_0 = -0.04x_0^2 + 1.2x_0 \)

\[ y_0 = -0.04(35.307)^2 + 1.2(35.307) \]
\[ y_0 = -7.494 \]

so the activation point is (35.307, -7.494)

For the second value is impossible.
The equation of the tangent is \( y = mx + b \)

\[ y = -0.295x + 13.977 \]
Part 3: A Long Flight

I want to try another level that the parabola path without activating a boost could reach. If we want to kill the 3rd pig from the left at point (40,5) by boosting through the glass, what should we do? This time we try to go as high as we could. Each level has a different weak spot on the pigs’ fort. For this level at the top of the fort are thin glasses. At higher altitude, there are more chances for the bird’s path after the boost striking from the top of the fort.

Choose 10 points, find the coordinate to make an equation.

<table>
<thead>
<tr>
<th>Trial #</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0 1 2 3 5 7 8 10 13 15</td>
</tr>
<tr>
<td>y</td>
<td>0 7 13 18 25 28 28 25 13 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Trial #</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0 4 6 7 8 9 11 12 14 15</td>
</tr>
<tr>
<td>y</td>
<td>0 22 27 28 28 27 22 18 7 0</td>
</tr>
</tbody>
</table>
Plot down the dot and find the line of best fit on the “demos” app (you have to pay for it). Click find the line of best fit. It also gave us a vertex. But we don’t need it because we got the equation with the correlation 1.00.

The equation is \( y = -0.5 x^2 + 7.5x \)

Find the derivative with method #2;

\[
\begin{align*}
  f^1(x) &= ab(x^{b-1}) + cd(x^{d-1}) \\
  f^1(x) &= 2(-0.5)(x^{2-1}) + 1(7.5)(x^{1-1}) \\
  f^1(x) &= -x + 7.5
\end{align*}
\]

Use The Tangent Line Function to solve.

\[
\begin{align*}
  &y - y_0 = f^1(x_0) \cdot (x - x_0) \\
  &5 - (-0.5 x_0^2 + 7.5x_0) = (-x_0 + 7.5) \cdot (40 - x_0) \\
  &5 + 0.5 x_0^2 - 7.5x_0 = -40x_0 - 7.5x_0 + 300 + x_0^2 \\
  &0 = 0.5 x_0^2 - 40x_0 + 295
\end{align*}
\]

Solve for x-intercepts

By using the quadratic formula: 
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
a = 0.5 , \quad b = -40 \quad \text{and} \quad c = 295
\]

\[
x = \frac{40}{2(0.5)} \pm \frac{\sqrt{(-40)^2 - 4(0.5)(295)}}{2(0.5)}
\]

\[
x = 40 \pm 31.780
\]

x-intercept are 8.22 and 71.78

sub \( x_0 \) in the \( f(x_0) = y_0 = -0.5x_0^2 + 7.5x_0 \)

\[
y_0 = -0.5(-8.22)^2 + 7.5(-8.22) = 27.866
\]

so the activation point is (8.22, 27.866)

sub \( x_0 \) in the \( f(x_0) = y_0 = -0.5x_0^2 + 7.5x_0 \)

\[
y_0 = -0.5(-71.78)^2 + 7.5(-71.78) = -1992.834
\]

so the activation point is (71.78, -1992.834)

For the second value is impossible due to the negative value of \( y \).

The equation of the tangent is \( y = mx + b \)

\[
y = (-x + 7.5)x + b
\]

\[
y = (- (8.22) + 7.5)x + b
\]

\[
5 = -0.72(40) + b
\]

\[
b = 33.8
\]

\text{equation is} \quad y = -0.72x + 33.8
The bird will strike the pig
Part 4: Conclusion

There is always a possible way to beat the game angry bird even thought the level is super hard. If there is a grid on the game every level will be easy to calculate where to hit. I tested 4 levels that were rated very hard by the players. I had to play it in the computer so I could run the program that shows the graph grid on the screen while I was playing. So am able to calculate the path before every launch of any birds in the game. I was able to eliminate all the pigs and got 3 stars on 1st or 2nd try. The calculating method is very effective on a hard level. There is no time waste for a retry which we would not know how many retry we will be ending up taking. I should have done this 3 years ago, so I did not have to waste my time retried hard levels more than 20 times on average.

Part 5: Reflection

I love playing games. I used to addict to the game angry bird even though I stop playing it but a lot of people do, people were looking for cheats. But instead of using cheats. I want to be more productive by applying math to the game. From this project, I have realized that I could apply higher level of math to a lot of things around me in everyday lives. Because all those things were composed of math in themselves. And there are so many programs out there that are able to help me achieve that. But the program is very raw, the way to change the settings of the grid needs codes to do because there is no control panel. My coding skill went from nothing to something after that. This project also improving my skill of problems solving due to a lot of elements went wrong during the coding process and calculating process.

In the future, I would like to construct every model for every birds in the games even though it will be a lot harder than this model I just made but it would be useful to the players especially the young ones, and answer a lot of people’s question “How could we get to use math in everyday life” by showing them how. In the future I want be either actuary or engineer. I have to be able to apply math to everyday life’s situation. From doing this projects, I was introduced with many new ways and tools I could use to make that happen.
Part 6: Reference


Try to test the graph - https://www.desmos.com/calculator

My dad, Poonwong Vongsathorn